UDC 532.51

ON THE NONLINEAR THEORY OF STABILITY OF PERIODIC FLOWS

PMM Vol. 36, №2, 1972, pp. 263-271

V. I. KLIATSKIN
(Moscow)

(Received June 25, 1971)

The nonlinear problem of two-dimensional motion of an unbounded incompressible viscous fluid under the action of a spatially periodic force is considered in a finite-dimensional approximation. When the number of finite-amplitude perturbation harmonics is restricted, a stationary solution is found to exist, corresponding to a secondary flow. This solution is however unstable with respect to smaller perturbations. Equations expressing the total contribution of the most unstable harmonics are derived and are found to have no stationary solutions.

Papers [4 - 7] propose a relatively simple approximate procedure of computing secondary equilibrium flows for the purpose of describing the first stage of formation of the finite-amplitude perturbations according to the scheme given in [1] (also see [2 and 3]). In this procedure the Reynolds stresses governed by the finite-amplitude perturbations are balanced against the dissipation and the mean flow.

The problem of the secondary flows, stationary or periodic, arising when the laminar flows of a viscous incompressible fluid become unstable, is considered in [8 - 13].

The concept of mechanical hydrodynamic systems embracing finite-dimensional approximations to the equations of hydrodynamics, is introduced in [14]. In [15] a mechanical model is constructed in the form of chains of simplest systems. This model imitates the cascade mechanism of the energy transfer in a developed turbulent flow and serves as an illustration of a possible simplest realisation of the Landau scheme.

Below an equilibrium mode of a secondary flow is studied in a finite-dimensional approximation for a two-dimensional case.

1. We consider a two-dimensional motion of an incompressible viscous fluid in the xy-plane under the action of a spatially periodic force acting in the direction of the x-axis and equal to $\gamma \sin py$ $(\gamma > 0)$.

The Navier-Stokes and continuity equations have a stationary solution corresponding to a laminar flow moving in the x-direction with a velocity $p^{-2}\nu^{-1}\gamma \sin py$ under a constant pressure (ν is the kinematic viscosity coefficient). As was shown in [11, 16] using the linear formulation, this solution is unstable with respect to small perturbations.

Introducing the characteristic length p^{-1}, velocity $p^{-2}\nu^{-1}\gamma$ and time $p^{-1}\nu^{-1}\gamma$ and passing to the dimensionless variables, we can write the equations of motion and continuity in the form

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{R}\Delta u + \frac{1}{R}\sin y \qquad \left(R = \frac{\gamma}{\nu^2 p^3}\right)$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{R}\Delta v, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1.1)$$

where u and v are the x- and y-velocity components, P is the dimensionless pressure and R is the Reynolds' number. For the stream function we have

$$\left(\frac{\partial}{\partial t}-\frac{\Delta}{R}\right)\Delta\psi-\frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x}=\frac{1}{R}\cos y \quad (1.2)$$
$$(u=\partial\psi/\partial y,\quad v=-\partial\psi/\partial x)$$

The infinitesimal perturbations grow rapidly in time by virtue of their linear instability. This in turn causes the growth of the Reynolds' stresses which are described by the nonlinear terms of (1.1) and the amplitude of the laminar flow diminishes until another, different equilibrium flow is established.

Let us represent the hydrodynamic fields in the form

$$u=U(y,\ t)+u'(x,\ y,\ t),\qquad v=v'(x,y,t)$$
$$P=P_0+P'(x,\ y,\ t),\quad \psi=\Psi(y,\ t)+\psi'(x,\ y,\ t) \quad (1.3)$$

Here $U(y,\ t)$ is the new equilibrium flow profile to be determined together with the Reynolds' equilibrium stresses. The corresponding finite perturbations are denoted by a prime. We assume the perturbations to be harmonic in x, their wavelength equal to $2\pi/\alpha\ (\alpha>0)$, and during the first stage we shall only be concerned with the non-linear interaction between the first perturbation harmonic and the average flow, disregarding the higher harmonics, their mutual interaction and their interaction with the averaged flow. In this case the average flow (averaged with respect to x over a single wavelength) is defined by $U(y,\ t)$.

Let us write the perturbations in the form

$$\varphi'(x,\ y,\ t)=\varphi^{(1)}(y,\ t)\exp(i\alpha x)+\varphi^{(-1)}(y,\ t)\exp(-i\alpha x)$$
$$(\varphi'=u',\ v',\ P',\ \psi')$$

where $\varphi^{(-1)}$ is a complex conjugate of $\varphi^{(1)}$. Then (1.1) yields the following system of equations describing the average flow and the perturbations $v^{(1)}$ (after eliminating P' and u')

$$\frac{\partial}{\partial t}U+\frac{i}{\alpha}\left(v^{(-1)}\frac{\partial^2v^{(1)}}{\partial y^2}-v^{(1)}\frac{\partial^2v^{(-1)}}{\partial y^2}\right)=\frac{1}{R}\frac{\partial^2U}{\partial y^2}+\frac{1}{R}\sin y$$
$$\left(\frac{\partial}{\partial t}-\frac{\Delta}{R}\right)\Delta v^{(1)}+i\alpha\left[U\Delta v^{(1)}-v^{(1)}\frac{\partial^2U}{\partial y^2}\right]=0 \quad (1.4)$$

2. Let us dwell on the results of the linear theory of stability. The stationary solution of the linear problem corresponds to the flow profile $U(y)=\sin y$. The equation for the perturbations $v^{(1)}$ obtained here from the second equation of (1.4) was studied in [11, 16] where it was shown that for the perturbations of the type

$$v^{(1)}=\sum_{n=-\infty}^{\infty}v_n^{(1)}\exp\{\sigma t+iny\} \quad (2.1)$$

2π-periodic in y, with certain restrictions imposed on the wave number α and on R, real values of σ exist, i.e. the solutions are unstable. The dispersion equation for σ appears in the form of an infinite continued fraction. When $\alpha\to0$ [11], the critical Reynolds number $R_*=\sqrt{2}$.

Let us limit ourselves to a finite number of harmonics in y. In this case the dispersion equation for σ has the form corresponding to the convergent of the general continued fraction obtained in [16], and the critical Reynolds number $R_*=\sqrt{2}$ with $\alpha\to0$ in accordance with the result of [11]. Thus the large wavelength perturbations moving along the x-axis show greater instability. We can therefore assume that $\alpha \ll 1$. In this case the components of the eigenvector $\{v_n^{(1)}\}$ beginning with $n = \pm 2$ and higher, have the order of at least α^4.

A solution of the second equation of (1.4) with $U = \sin y$ can be sought on other classes of functions. For example, if the solution is sought on the class of functions 4π-periodic in y, then we obtain for the harmonics of the form

$$v^{(1)} = \sum_{n=-\infty}^{\infty} v_{n+1/2}^{(1)} \exp\left\{\sigma t + i\,\frac{2n+1}{2}\,y\right\} \tag{2.2}$$

a system of equations containing no harmonics of different type. Limiting ourselves to a minimum number of harmonics $(v_{\pm 3/2}^{(1)}, v_{\pm 1/2}^{(1)})$ we find that the solution of this system also corresponds to an unstable mode, that $R_* \approx 6.5$ and that the most unstable perturbations are those with the wave number $\alpha \approx 0.44$. The eigennumber σ and the eigenvector $\{v^{(1)}\}$ are complex and this leads to a secondary flow which is both spatially and temporally periodic.

3. Let us now investigate the system (1.4). We consider the perturbations of the type (2.1) as being the most unstable. Assuming that $\alpha \ll 1$ corresponds to the most unstable perturbations and, that $\sigma_{\pm 2}^{(1)}$ and the smaller harmonics are of higher order in α we can limit our considerations to the harmonics $v_0^{(1)}$ and $v_{\pm 1}^{(1)}$. In this case

$$J(y, t) = U(t)\sin y \tag{3.1}$$

and the equation for $v^{(1)}$ becomes

$$\left(\frac{\partial}{\partial t} - \frac{\Delta}{R}\right)\Delta v^{(1)} + i\alpha U(t)\sin y\,[v^{(1)} + \Delta v^{(1)}] = 0 \tag{3.2}$$

Inserting (3.1) into (1.4) and utilizing the expansion (2.1) for $v^{(1)}$ we obtain for the functions

$$z_0 = v_0^{(1)}, \qquad z_+ = v_1^{(1)} + v_{-1}^{(1)}, \qquad z_- = (v_1^{(1)} - v_{-1}^{(1)})/2 \tag{3.3}$$

the following system of equations:

$$\frac{d}{dt}U + \frac{4}{\alpha}z_0 z_- = \frac{1}{R} - \frac{1}{R}U, \qquad \frac{d}{dt}z_0 - \alpha U z_- = -\frac{\alpha^2}{R}z_0$$

$$\frac{d}{dt}z_- - \frac{\alpha}{2}U z_0 = -\frac{1}{R}z_-, \qquad \frac{d}{dt}z_+ = -\frac{1}{R}z_+ \tag{3.4}$$

The equation for z_+ splits off from the remaining equations, consequently the corresponding perturbations can only decay with time. The remaining three equations form the simplest hydrodynamic system [14, 15] equivalent to the dynamic description of the motion of a gyroscope with anisotropic friction excited by the moment of external forces, taken with respect to the unstable axis. The stationary solution of (3.4) corresponds to the average flow profile and to the equilibrium Reynolds' stresses

$$U = \frac{\sqrt{2}}{R}, \qquad \frac{4}{\alpha}z_0 z_- = \frac{R-\sqrt{2}}{R^2}, \qquad z_+ = 0 \tag{3.5}$$

$$[v_0^{(1)}]^2 = \frac{R-\sqrt{2}}{2\sqrt{2}R^2}, \qquad v_1^{(1)} = \frac{\alpha}{\sqrt{2}}v_0^{(1)}, \qquad \alpha \ll 1, \qquad R > \sqrt{2}$$

or, in the dimensional quantities,

$$U(y) = \sqrt{2}\,\nu p \sin py, \qquad \langle u'v'\rangle = -\frac{\gamma}{p}\frac{R-\sqrt{2}}{R}\cos py \tag{3.6}$$

We note that the amplitude of the steady average equilibrium flow is independent of the amplitude of the excitation force.

The stream function of the steady equilibrium flow has the form

$$\psi_1 = -\frac{\sqrt{2}}{R}\cos y - \frac{2}{\alpha}\, v_0^{(1)}\, [\sqrt{2}\,\alpha \sin y \cos \alpha x + \sin \alpha x] \tag{3.7}$$

We also note that a random element exists in the steady flow, namely, the quantity $v_0^{(1)}$ may either be positive or negative depending on the signs of the amplitudes of the initial infinitesimal perturbations.

The corresponding streamlines of the flow at

$$R = 2R_* = 2\sqrt{2}\,(v_0^{(1)} > 0)$$

are described by the equation

$$\alpha \cos y + \sqrt{2}\alpha \sin y \cos \alpha x + \sin \alpha x = C$$

Figure 1 shows schematically the streamlines and the average flow profile.

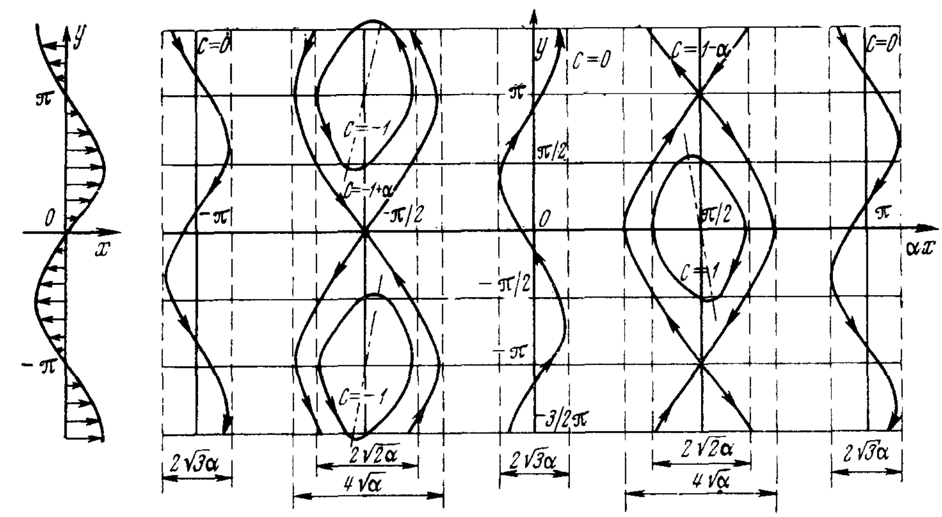

Fig. 1.

When the perturbations are sought on the class of functions (2.2), the resulting hydrodynamic system is of higher order with respect to the complex quantities $v_k^{(1)}$. An additional harmonic is generated in the average flow profile, and the interaction of the harmonics between themselves and with the average flow which are related to the energy transfer between different orders of magnitude, is supplemented with another interaction leading to an oscillatory energy transfer between the harmonics of the same order of magnitude. This yields a secondary flow periodic in time.

4. Let us investigate the stability of the secondary flow (3.7). Linearizing (1.2) with respect to the flow (3.7), we obtain the following equation for the perturbations

$$\left(\frac{\partial}{\partial t} - \frac{\Delta}{R}\right)\Delta\psi' + (u_1\Delta - \Delta u_1)\frac{\partial\psi'}{\partial x} + (v_1\Delta - \Delta v_1)\frac{\partial\psi'}{\partial y} = 0$$

$$(\psi = \psi_1 + \psi', \quad u_1 = \partial\psi_1/\partial y, \quad v_1 = -\partial\psi_1/\partial x) \tag{4.1}$$

We shall seek the solution of (4.1) on the class of functions which are $2\pi/\alpha$-periodic in x and 2π-periodic in y, i.e. we shall write the solution in the form

$$\psi' = \sum_{n,k=-\infty}^{\infty} C_n^k \exp\{\sigma t + i(ny + \alpha kx)\}, \qquad \bar{C}_n^k = C_{-n}^{-k}, \qquad C_0{}^\circ = 0 \qquad (4.2)$$

Inserting (4.2) into (4.1) we obtain the following system of equations for the coefficients C_n^k :

$$\Big\{iv_1^{(1)}(n-k)[n(n-2)+\alpha^2 k(k-2)]\,C_{n-1}^{k-1} + \frac{\alpha k}{\sqrt{2}R}[n(n-2)+\alpha^2k^2]\,C_{n-1}^{k} -$$
$$- iv_1^{(1)}(n+k)[n(n-2)+\alpha^2 k(k+2)]\,C_{n-1}^{k+1}\Big\} +$$
$$+\Big\{inv_0^{(1)}[n^2+\alpha^2k(k-2)]\,C_n^{k-1} + \Big(\sigma + \frac{n^2+\alpha^2k^2}{R}\Big)(n^2+\alpha^2k^2)\,C_n{}^k +$$
$$+ inv_0^{(1)}[n^2+\alpha^2k(k+2)]\,C_n^{k+1}\Big\} +$$
$$+\Big\{-iv_1^{(1)}(n+k)[n(n+2)+\alpha^2k(k-2)]\,C_{n+1}^{k-1} - \frac{\alpha k}{\sqrt{2R}}[n(n+2)+$$
$$+\alpha^2k^2]\,C_{n+1}^{k} + iv_1^{(1)}(n-k)[n(n+2)+\alpha^2k(k+2)]\,C_{n+1}^{k+1}\Big\} = 0 \qquad (4.3)$$

The quantities $v_1^{(1)}$ and $v_0^{(1)}$ appearing here are defined by (3.5). If we assume that the solution (3.7) is accurate up to and including α^2 terms, we shall neglect the α^3 and higher terms in the course of solving (4.3).

We see that the flow described by the stream function (3.7) is stable if we limit ourselves in the system (4.3) to the harmonics $n = 0, \pm 1$ and $k = 0, \pm 1$ (i.e. if the solution of (4.1) is sought in the form (3.7)). It follows that the flow (3.7) can be unstable only with respect to smaller perturbations. Let us limit ourselves to the harmonics $n = 0, \pm 1, \pm 2;\ k = 0, \pm 1, \pm 2$ and take into account only the principal (in α) terms, neglecting the α^3 terms. It can easily be shown from (4.3) that all harmonics in y beginning with $n = \pm 2$ and smaller, will be of at least the α^3-order and can therefore be neglected in the approximation considered.

Let us replace the amplitudes $C_n{}^k$ of the harmonics with their linear combinations

$$C_{-1}^k + C_1^k = z_+^k, \qquad C_{-1}^k - C_1^k = z_-^k, \qquad C_0^k = z_0^k \qquad (4.4)$$

Then the system (4.3) splits into two seventh order systems for z

$$\left\|\begin{array}{ccccc:cc}
\sigma+\frac{1}{R} & -iv_0^{(1)} & 0 & 0 & 0 & -2iv_1^{(1)} & 0\\
-iv_0^{(1)} & \sigma+\frac{1}{R} & -iv_0^{(1)} & 0 & 0 & -\frac{\sqrt{2}\alpha}{R} & 0\\
0 & -iv_0^{(1)} & \sigma+\frac{1}{R} & -iv_0^{(1)} & 0 & -2iv_1^{(1)} & 2iv_1^{(1)}\\
0 & 0 & -iv_0^{(1)} & \sigma+\frac{1}{R} & -iv_0^{(1)} & 0 & \frac{\sqrt{2}\alpha}{R}\\
0 & 0 & 0 & -iv_0^{(1)} & \sigma+\frac{1}{R} & 0 & 2iv_1^{(1)}\\
\hdashline
3iv_1^{(1)} & -\frac{\alpha}{\sqrt{2}R} & -iv_1^{(1)} & 0 & 0 & \sigma+\frac{\alpha^2}{R} & 0\\
0 & 0 & iv_1^{(1)} & \frac{\alpha}{\sqrt{2}R} & -3iv_1^{(1)} & 0 & \sigma+\frac{\alpha^2}{R}
\end{array}\right\| \times$$

/ cont.

$$\times \left\| z_+^{-2} \quad z_-^{-1} \quad z_+^{0} \quad z_-^{1} \quad z_+^{2} \quad z_0^{-1} \quad z_0^{1} \right\|' = 0 \tag{4.5}$$

$$\left\| \begin{matrix} \sigma + \frac{4\alpha^2}{R} & 0 & -\frac{\sqrt{2}\,\alpha}{R} & 0 & 0 & 0 & 0 \\ 0 & \sigma + \frac{4\alpha^2}{R} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}\,\alpha}{R} \\ -\frac{\sqrt{2}\,\alpha}{R} & 0 & \sigma + \frac{1}{R} & -iv_0^{(1)} & 0 & 0 & 0 \\ -4iv_1^{(1)} & 0 & -iv_0^{(1)} & \sigma + \frac{1}{R} & -iv_0^{(1)} & 0 & 0 \\ 0 & 0 & 0 & -iv_0^{(1)} & \sigma + \frac{1}{R} & -iv_0^{(1)} & 0 \\ 0 & 4iv_1^{(1)} & 0 & 0 & -iv_0^{(1)} & \sigma + \frac{1}{R} & -iv_0^{(1)} \\ 0 & \frac{2\sqrt{2}\,\alpha}{R} & 0 & 0 & 0 & -iv_0^{(1)} & \sigma + \frac{1}{R} \end{matrix} \right\| \times$$

$$\times \left\| z_0^{-2} \quad z_0^{2} \quad z_-^{-2} \quad z_+^{-1} \quad z_-^{0} \quad z_+^{1} \quad z_-^{2} \right\|' = 0 \tag{4.6}$$

In the course of deriving (4.5) and (4.6) we have omitted in the expressions of the form $-iv_0^{(1)}(1-\alpha^2)$, $[\sigma + (1+\alpha^2)/R](1+\alpha^2)$ etc., the terms of the α^2-order. This is connected with the fact that the eigenvalues σ for the systems (4.5) and (4.6) which can correspond to instability of the flow (3.7) are themselves of the α^2-order. The neglected terms do not contribute anything towards the value of σ in this approximation and only become apparent in the higher orders (in α) which, as was shown before, were neglected. For this reason, we must assume in the case of unstable solutions of (4.5) and (4.6) that in the first five equations of (4.5) and in the last five equations of (4.6) $\sigma = 0$.

Let us consider the system (4.5). The system (4.6) can be treated in exactly the same way, and the eigenvalues σ for this system correspond to the stability of the flow (3.7) at all $R > R_*$. Multiplying (4.5) by the inverse of the submatrix marked out with a broken line, and separating the solution into its real and imaginary parts, we obtain the following two systems of equations:

$$\left\| \begin{matrix} 1 & 0 & -\frac{2\sqrt{2}\,\alpha v_0^{(1)}}{R\Delta_5}\left[1 + \frac{(v_0^{(1)})^2}{R^2}\right] \\ 0 & 1 & \frac{\sqrt{2}\,\alpha}{R\Delta_5}\left[3(v_0^{(1)})^4 - \frac{2(v_0^{(1)})^2}{R^2} - \frac{1}{R^4}\right] \\ -3v_1^{(1)} & -\frac{\alpha}{\sqrt{2}R} & \sigma + \frac{\alpha^2}{R} \end{matrix} \right\| \left\| \begin{matrix} \operatorname{Im} z_+^{-2} \\ \operatorname{Re} z_-^{-1} \\ \operatorname{Re} z_0^{-1} \end{matrix} \right\| = 0 \tag{4.7}$$

$$\left\| \begin{matrix} 1 & 0 & 0 & \frac{2\sqrt{2}\,\alpha v_0^{(1)}}{R\Delta_5}\left[\Delta_3 - \frac{(v_0^{(1)})^2}{R}\right] \\ 0 & 1 & 0 & \frac{\sqrt{2}\alpha}{R\Delta_5}\left[3(v_0^{(1)})^4 + \frac{2(v_0^{(1)})^2}{R^2} - \frac{1}{R^4}\right] \\ 0 & 0 & 1 & \frac{4\sqrt{2}\,\alpha v_0^{(1)}\Delta_3}{R^2\Delta_5} \\ 3v_1^{(1)} & -\frac{\alpha}{\sqrt{2}R} & -v_1^{(1)} & \sigma + \frac{\alpha^2}{R} \end{matrix} \right\| \left\| \begin{matrix} \operatorname{Re} z_+^{-2} \\ \operatorname{Im} z_-^{-1} \\ \operatorname{Re} z_+^{0} \\ \operatorname{Im} z_0^{-1} \end{matrix} \right\| = 0 \tag{4.8}$$

where

$$\Delta_n = \frac{1}{R}\Delta_{n-1} + (v_0^{(1)})^2 \Delta_{n-2} \qquad \left(\Delta_1 = \frac{1}{R}, \qquad \Delta_2 = \frac{1}{R^2} + (v_0^{(1)})^2\right)$$

The eigenvalue σ corresponding to the system (4.7) has the form

$$\sigma = \frac{\alpha^2}{R^2\Delta_5}\left[12\,(v_0^{(1)})^4 + \frac{4\,(v_0^{(1)})^2}{R^2}\right] \geqslant 0 \quad \text{for} \quad R \geqslant \sqrt{2} \tag{4.9}$$

and this corresponds to the instability of the flow (3.7) at all $R > R_*$. The eigenvalue σ for (4.8) on the other hand, corresponds to the stability of the flow (3.7) at all $R > R_*$.

Let us now consider the nonlinear equation (1.2) describing the stream function, taking into account not only the interaction of the perturbations with the average stream, but also the nonlinear interactions between the perturbations themselves (the harmonics discussed in Sect. 3 and 4). For $R > R_*$ we arrive at a chain of two systems of equations of the type (3.4), in which the stationary solution of (3.4) generates not only the smaller scale motions, but also motions of the same order of magnitude. In this case the stream function (3.7) is supplemented by the following additional terms

$$2\,\mathrm{Re}\, z_0^{-1}\cos\alpha x + \frac{2}{\alpha}\,\mathrm{Im}\, z_+^{-2}\cos y \sin 2\alpha x - \frac{2}{\alpha}\,\mathrm{Re}\, z_-^{-1}\sin y \sin \alpha x$$

At the same time the degree of randomness of the solution increases as compared with (3.7), the effect connected with the sign of the amplitudes of the smaller scale infinitesimal perturbations.

Thus we find that in this case the nonlinear interaction between the infinitesimal perturbations and the interaction of the perturbations with the average flow are both significant. We can no longer limit ourselves to a finite number of harmonics in x (this, in effect, represents the Galerkin method with trigonometric monomials as coordinate functions) and must consider their infinite series. At the same time the harmonics in the y-direction may be restricted to those with $n = \pm 1, 0$.

5. Let us write the stream function in the form

$$\psi(x, y, t) = \psi_{-1}(x, t)\exp(-iy) + \psi_0(x, t) + \psi_1(x, t)\exp(iy) \quad (\overline{\psi}_{-1} = \psi_1) \tag{5.1}$$

where $\psi_i(x, t)$ are $2\pi/\alpha$-periodic functions in x.

Inserting (5.1) into (1.2), neglecting the α^3 terms in the interactions between the harmonics and the α^2 terms in the dissipative terms for the harmonics $\psi_{\pm 1}$ and introducing new functions $\psi_{-1} + \psi_1 = 2\psi_+$ and $\psi_{-1} - \psi_1 = 2i\psi_-$, we obtain

$$\left(\frac{\partial}{\partial t} + \frac{1}{R}\right)\psi_+ - \psi_-\frac{\partial\psi_0}{\partial x} = -\frac{1}{2R}, \qquad \left(\frac{\partial}{\partial t} + \frac{1}{R}\right)\psi_- + \psi_+\frac{\partial\psi_0}{\partial x} = 0$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{R}\frac{\partial^2}{\partial x^2}\right)\psi_0 + 2\left(\psi_-\frac{\partial\psi_+}{\partial x} - \psi_+\frac{\partial\psi_-}{\partial x}\right) = 0 \tag{5.2}$$

This system of equations sums the infinite series of harmonics in x and represents a generalization of the system (3.4) of gyroscopic type equations to the infinite case. Its distinguishing feature is the absence of stationary solutions periodic in x (except the solution corresponding to a laminary flow), unlike the systems with a finite number of harmonics in x, which all have stationary solutions.

The author thanks A. M. Obukhov for his interest and valuable comments.

BIBLIOGRAPHY

1. Landau L. D., On the problem of turbulence. Dokl. Akad. Nauk SSSR, Vol. 44, №8, 1944.
2. Landau L. D. and Lifshits E. M., Mechanics of Continuous Media. M. Gostekhizdat, 1954.
3. Monin A. S. and Iaglom A. M., Statistical Hydromechanics. Pt. 1, M., "Nauka", 1965.
4. Stuart J. T., On the non-linear mechanics of hydrodynamic stability. J. Fluid Mech. Vol. 4, №1, 1958.
5. Stuart J. T., On the non-linear mechanics of wave disturbances in stable and unstable parallel flows, Pt. 1, J. Fluid Mech., Vol. 9, №3, 1960.
6. Watson J., On the nonlinear mechanics of wave disturbances in stable and unstable parallel flows, Pt. 2, J. Fluid Mech., Vol. 9 №3, 1960.
7. Reynolds W. C. and Potter M. C., Finite-amplitude instability of parallel shear flows. J. Fluid Mech., Vol. 27, №3, 1967.
8. Brushlinskaia N. N., Limit cycles of the equations of motion of a rigid body and the Galerkin equations of hydrodynamics. Dokl. Akad. Nauk SSSR, Vol. 157, №5, 1964.
9. Brushlinskaia N. N., Behavior of the solutions of the hydrodynamic equations during the passage of the Reynolds number through its critical value. Dokl. Akad. Nauk SSSR, Vol. 162, №4, 1965.
10. Brushlinskaia. N. N., Generation of a periodic solution and a torus from a laminar flow. In coll. "Some Problems of Mechanics of Rocks"., M., 1968.
11. Iudovich V. I., Example of the generation of the secondary stationary or periodic flow when there is loss of stability of the laminar flow of a viscous incompressible fluid. PMM Vol. 29, №3, 1965.
12. Iudovich V. I., An example of loss of stability and generation of a secondary flow in a closed vessel. Matem. sb., Vol. 74(116), №4, 1967.
13. Iudovich V. I., The onset of auto-oscillations in a fluid. PMM Vol. 35, №4, 1971.
14. Obukhov A. M., On integral invariants in systems of hydrodynamic type. Dokl. Akad. Nauk SSSR, Vol. 184, №2, 1969.
15. Obukhov A. M., On certain general features of the atmosphere dynamics equations. Izv. Akad. Nauk SSSR, Fiz. atmosfery i okeana, Vol. 7, №7, 1971.
16. Meshalkin L. D. and Sinai Ia. G., Investigation of the stability of a stationary solution of a system of equations for the plane motion of an incompressible viscous liquid. PMM Vol. 25, №6, 1961.

Translated by L. K.